ESTIMATION OF FORCE OF MORTALITY FROM THIRD DEGREE POLYNOMIAL OF l_x

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Abstract

Force of mortality is an effective tool for indicating mortality level for a population which is an instantaneous measure of probability of death in a particular instant given survival up to that time. As the force of mortality is a function of the life table number of survivors at a particular age x; in this study third degree polynomial model is fitted to estimate the force of mortality using life table of Matlab demographic surveillance area for both sexes respectively. With more than 99 percent stability of the fitted models, pattern of force of mortality illustrated that; after a certain age (around 55 years) the force of mortality increase sharply to the remaining age and after age 15 the force of mortality is lower for females compared to males for Matlab HDSS.

Keywords: Mathematical modeling; Force of mortality; Matlab HDSS

Introduction

Various mortality rates are used for analyzing and evaluating population and health agendas and strategies (Sen, 1998). Infant and child mortality rates are a significant indicator of a country's level of socioeconomic development and quality of life. The mortality profile of a country, including the causes of deaths, pattern in different age-groups along with gender; provides a basis for policy-making that will eventually lead to a decrease in unwarranted deaths. Bangladesh addresses the issues and strives hard to meet the challenges of lowering the death rates due to various causes.

Information on mortality completely depends on censuses as well as other sources by means of some complicated indirect techniques. Bangladesh has not started complete Vital Registration System (VRS) yet, though there are some experimental systems under the Bangladesh Bureau of Statistics (BBS). Various demographic surveys funded by the government and foreign-AIDS, procure data on a sampling basis but they do not provide sufficient information on mortality data (BDHS-2007).

A number of good research works on infant mortality and child mortality are conducted (Chowdhury M.E. et al 2007; Rahman and Sarkar, 2009). But, overall scenarios of mortality are studied in a very limited scale in Bangladesh. Recent works showed decreasing trend in mortality measures. Application of indirect estimation on the abridged life table for both sexes showed a traditional U-shaped pattern for ASDRs; where life tables have been constructed using UN model life table for South Asia (Islam, 2007). In another study, female adult mortality rates have been estimated using life tables constructed from male widowed information of Bangladesh (Islam, 2006). Decreasing trend is observed also in that study for age specific mortality rates.

Besides traditional mortality rates, force of mortality is another effective mortality indicator for a population. The force of mortality is the probability of death in a particular instant given survival up to that time. This is an instantaneous measure, rather than an interval measure. Force of mortality is defined as a function of the number of survivors in a particular age group of a decrement life table. It is to be noted here that a life table is very elegant mathematical tools in Demography to follow up the mortality or fertility scenario for a cohort or whole population. Life table is far and wide used to estimate various demographic parameters such as IMR, CDR, ASDRs, NRR, survival rate, replacement index. However, the estimation of force of mortality is a laborious job to do as there are no specific methods to construct mathematical models for a number of survivors associated with age. There are few solutions using established distribution, but those are also very fuzzy to estimate the force of mortality (McCutcheon, 1983).

Thus, to have a full profile of mortality patterns in Bangladesh; the aim of the current study is to estimate the force of mortality in Bangladesh considering different age groups and both genders. To estimate the force of mortality, abridged life table obtained from a demographic surveillance area is used and a mathematical model is fitted for the number of survivors in the age group of life table associated with age.

Data and Methodology

It is already mentioned in the previous section that Bangladesh has not started complete Vital Registration System yet. The only source of analyzing mortality data is census reports or UN population facts (United Nations, 2012). For current study the vital registration and maternal and child health data gathered from Matlab, Bangladesh, in 2010 is utilized. The data were collected by the Health and Demographic Surveillance System of ICDDR,B . Since 1966, the HDSS has maintained the registration of births, deaths, and migrations, in addition to carrying out periodical censuses. Registration of marital unions and dissolutions, internal movement, are recorded and published as annual report each year. Matlab HDSS is recognized worldwide by population experts as one of the long-term demographic surveillance sites in a developing country. The abridged life tables containing ASDR, number of survivors (lx) and expectancies of life for both sexes are taken from analysis of 2010 Matlab Health and Demographic Events (Matlab HDSS, 2012).

Generally, abridged life tables are constructed using the following mathematical relationships:

Number of survivors in a particular age x is l_x and number of death in an interval x+n is Number of deaths in a particular age x is, ${}_nd_x = l_x - l_{x+n}$

Probability of surviving in particular age x to x+n is, ${}_np_x$ and death is ${}_nq_x$ where ${}_np_x = 1 - {}_nq_x$ where ${}_nq_x = {}_nd_x / l_x$. Thus, in a time interval n, $l_{x+n} = l_x\, {}_np_x$ and ${}_nd_x = l_x\, {}_nq_x$

Number of person- year lived by the cohort is, ${}_nL_x = \int_0^n l_{(x+t)}\,.dt$ which is equivalent to

$${}_nL_x = \frac{n\,(l_x + l_{x+n})}{2};\ (x \geq 2)$$

For x<2; $L_0 = 0.201\, l_0 + 0.8\, l_1$ and $L_1 = 0.410\, l_1 + 0.590\, l_2$

Number of person-year lived by the life table population is $T_x = \int_0^\infty L_{(x+t)}\,.dt$ which is equivalent to

$$T_x = \sum_{t=0}^{\infty} L_{(x+t)}$$

Expectancies of life at age x is, $e_x = \frac{T_x}{L_x}$

Force of mortality and modeling of l_x

The force of mortality at age x is defined as the instantaneous probability of death at age x which is a function of number of survivors (l_x) at age x; symbolically,

$$\mu_x = \lim_{t\to 0} \frac{l_{(x+t)} - l_x}{t\,.l_x} = -\frac{1}{l_x}\,.\frac{d}{dx}\,(l_x) = -\frac{d}{dx}\,(\ln l_x)$$

It appears from the scattered plot of the number of persons surviving at an exact age x (l_x) for both sexes of Matlab HDSS 2010 by age groups that l_x can be distributed by polynomial model for different ages rather than a linear model. Hence we tried to fit an nth degree polynomial instead of linear regression model. The structure of the model is,

$$y = a_0 + \sum_{i=1}^{n} a_i \,.\, x^i + u \qquad (1)$$

Where, x is the exact age, y is the number of survivors at age x (l_x), a_i is the coefficients of the model (for i=1,2,3; a_0 is the constant term of the fitted model).

Model validation technique

Cross validity prediction power (CVPP); ρ^2_{CV}; is applied here to test the stability of the fitted model. Symbolically,

$$\rho^2_{CV} = 1 - \frac{(n-1)(n-2)(n+1)}{n(n-k-1)(n-k-2)}\,(1 - R^2)$$

Where 'n' is the number of observations, 'k' is the number of predictors in the model, 'R' is the correlation between observed and predicted values of the dependent variable. The shrinkage of the model is the absolute difference of CVPP and R^2. Moreover, the stability of R^2 of the model is defined as difference between 1 and shrinkage (Stevens, 1996).

Again, from (i),

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots\ldots\ldots + u$$

$$\Rightarrow y' = a_1 + 2a_2x + 3a_3x^2 + \ldots\ldots\ldots + na_nx^{(n-1)} \qquad (2)$$

Using (i) and (ii) in definition of force of mortality, we have

$$\mu_x = -\frac{d}{dx}(\ln l_x) = -\frac{1}{l_x}\cdot\frac{d}{dx}(l_x) = -\frac{(1a_1 + 2a_2x + 3a_3x^2 + \ldots\ldots + na_nx^{(n-1)})}{a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots\ldots + u} \qquad (3)$$

Results

The abridged life table for male and female for Matlab DHSS 2010 are presented in table 1 and 2 respectively.

Table 1: Abridged life table for male (Matlab HDSS 2010).

Age x	l_x	${}_nd_x$	${}_nq_x$	${}_np_x$	${}_nL_x$	T_x	e_x
0	100000	3290	0.032900000	0.9671000	97368	6926729	69.3
1	96710	311	0.003215800	0.9967842	96526	6829361	70.6
2	96399	258	0.002676376	0.9973236	96270	6732835	69.8
3	96141	156	0.001622617	0.9983774	96063	6636565	69.0
4	95985	112	0.001166849	0.9988332	95929	6540502	68.1
5	95873	253	0.002638908	0.9973611	478782	6444573	67.2
10	95620	241	0.002520393	0.9974796	477546	5965791	62.4
15	95379	387	0.004057497	0.9959425	476004	5488245	57.5
20	94992	763	0.008032255	0.9919677	473199	5012241	52.8
25	94229	408	0.004329877	0.9956701	470204	4539042	48.2
30	93821	969	0.010328178	0.9896718	466870	4068838	43.4
35	92852	639	0.006881920	0.9931181	462786	3601968	38.8
40	92213	1728	0.018739223	0.9812608	457073	3139182	34.0
45	90485	1964	0.021705255	0.9782947	447887	2682109	29.6
50	88521	2969	0.033540064	0.9664599	435728	2234222	25.2
55	85552	5972	0.069805498	0.9301945	413831	1798494	21.0

60	79580	9878	0.124126665	0.8758733	374618	1384663	17.4
65	69702	10661	0.152951135	0.8470489	323235	1010045	14.5
70	59041	13897	0.235378805	0.7646212	261675	686810	11.6
75	45144	13507	0.299198122	0.7008019	192653	425135	9.4
80	31637	13731	0.434017132	0.5659829	123392	232482	7.3
85	17906	17906	1.000000000	0.0000000	109090	109090	6.1

Table 2: Abridged life table for female (Matlab HDSS 2010).

Age x	l_x	${}_nd_x$	${}_nq_x$	${}_np_x$	${}_nL_x$	T_x	e_x
0	100000	2721	0.027210000	0.9727900	97824	7320542	73.2
1	97279	524	0.005386569	0.9946134	96970	7222718	74.2
2	96755	190	0.001963723	0.9980363	96660	7125748	73.6
3	96565	118	0.001221975	0.9987780	96506	7029088	72.8
4	96447	120	0.001244207	0.9987558	96387	6932582	71.9
5	96327	260	0.002699139	0.9973009	481035	6836195	71.0
10	96067	162	0.001686323	0.9983137	479961	6355160	66.2
15	95905	356	0.003712007	0.9962880	478704	5875199	61.3
20	95549	304	0.003181614	0.9968184	477045	5396495	56.5
25	95245	210	0.002204840	0.9977952	475743	4919450	51.7
30	95035	402	0.004230021	0.9957700	474248	4443707	46.8
35	94633	634	0.006699566	0.9933004	471704	3969459	41.9
40	93999	464	0.004936223	0.9950638	468926	3497755	37.2
45	93535	1231	0.013160849	0.9868392	464833	3028829	32.4
50	92304	811	0.008786185	0.9912138	459650	2563996	27.8
55	91493	3406	0.037226892	0.9627731	449570	2104346	23.0
60	88087	5331	0.060519713	0.9394803	428022	1654776	18.8
65	82756	9696	0.117163710	0.8828363	390956	1226754	14.8
70	73060	13719	0.187777169	0.8122228	332537	835798	11.4
75	59341	20864	0.351595019	0.6484050	244972	503261	8.5
80	38477	18489	0.480520831	0.5194792	144908	258289	6.7
85	19988	19988	1.000000000	0.0000000	113381	113381	5.7

The l_x values of table 1 and 2 are used to calculate the parameters of the models for male and female respectively. For both male and female, third degree polynomial fits the data well. The findings of the models are presented in table 3.

Table 3: Information on Model Fittings.

Model	n	k	R^2	F-statistic	p-value	ρ^2_{CV}	Shrinkage	Parameter	t-statistic	p-value
								a_0	147.235	0.0000
								a_1	-5.225	0.0001
								a_2	7.825	0.0000
(i)	22	3	0.99704	2020.72	0.0000	0.995668	0.001372	a_3	-14.597	0.0000
								a_0	95.743	0.0000
								a_1	-6.621	0.0000
								a_2	9.422	0.0000
(ii)	22	3	0.99096	658.03	0.0000	0.986769	0.0041908	a_3	-13.767	0.0000

The fitted model of l_x values for male of Matlab in 2010 is:

$$y = 97995.733720-478.246234x+21.6606x^2-0.322852x^3 \quad (4)$$

The fitted model of l_x values for female of Matlab in 2010 is:

$$y = 99812.375508-949.166312x+40.849428x^2-0.476912x^3 \quad (5)$$

Both of the models are highly cross-validated and their shrinkages are only 0.001372 and 0.0041908 respectively for male and female. Furthermore, both of the fitted models will be stable more than 99%. Besides, the parameters of the fitted models are highly significant; CVPP indicates 99% and 98% of variance are explained respectively for male and female. From t-statistics, it is found that all the parameters of the model are also highly significant. In both models, the stability of R^2 is more than 99%. The calculated value of F-test of the models are 2020.72322 and 658.02939 respectively for male and female with degrees of freedom (3, 18); whereas the corresponding tabulated value is only 5.09 at 1% level of significance. Therefore, the overall measure of the fitted models and its R^2 are highly significant.

The age-specific death rates (per thousand people) from HDSS 2010 report and our estimated force of mortality are presented in table 4. Though U-shaped pattern on age-specific death rates are almost sterilized due to low child mortality rates; still the presence of high infant mortality rates are present for both male and female. The pattern of force of mortality is decreasing up to 35 years in Matlab HDSS, but it has an increasing trend after the age 40years; rapidly increasing pattern is observed after the age interval 55 and above, that is, to infinity.

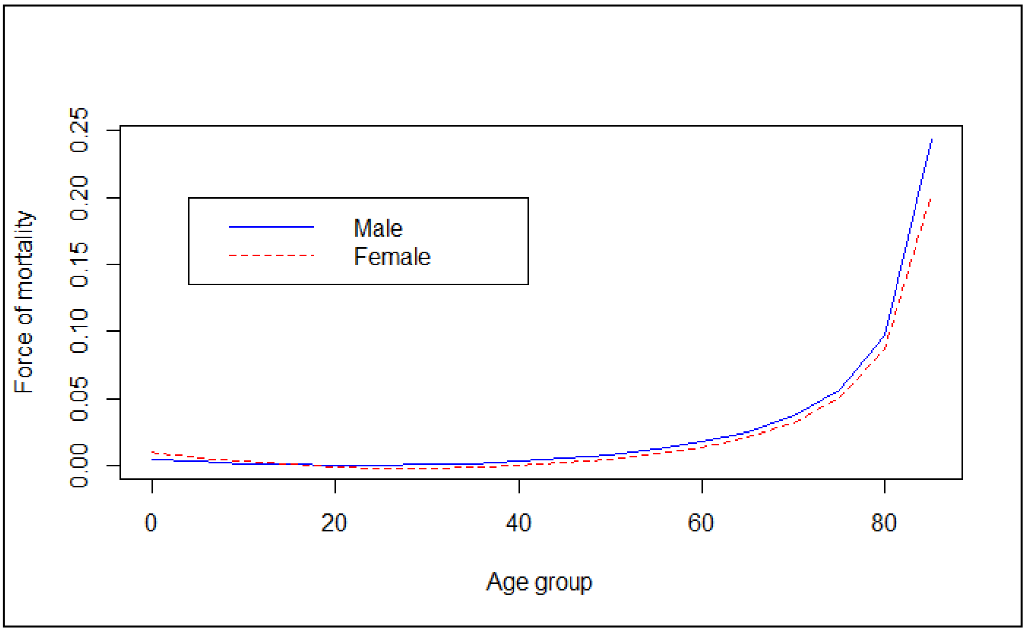
Table 4: Age-specific death rates (per thousand persons) and force of mortality for males and females (Matlab HDSS 2010).

Age x	Age-specific death rates (male)	Age-specific death rates (female)	Force of mortality (male)	Force of mortality (female)
0	32.9	27.2	0.004880276	0.0095095053
1	3.2	5.4	0.004468924	0.0087853056
2	2.7	2.0	0.004071917	0.0080703811
3	1.6	1.2	0.003690025	0.0073669233
4	1.2	1.2	0.003323958	0.0066770571
5	2.6	2.7	0.00.974374	0.0060028264
10	2.5	1.7	0.001492690	0.0029304269
15	4.1	3.7	0.0004899616	0.0004894886
20	8.0	3.2	7.992256e-06	7.902256e-06
25	4.3	2.2	5.963364e-06	5.863364e-06
30	10.3	4.2	5.328055e-04	4.928055e-04
35	6.9	6.7	0.001580488	0.0015689562
40	18.7	4.9	0.003177760	0.0003062654
45	21.7	13.2	0.005390802	0.0017637566
50	33.5	8.8	0.008347626	0.0046494459

55	69.8	37.2	0.01228084	0.0085341663
60	124.1	60.5	0.01761312	0.0137834645
65	153.0	117.2	0.02514985	0.0211153564
70	235.4	187.8	0.03657859	0.0320340924
75	299.2	351.6	0.05605122	0.0501635778
80	434.0	480.5	0.09712505	0.0867835349
85	1000.0	1000.0	0.2436403	0.2030147583

Estimated force of mortality against age is presented in figure 1, for both sexes. One of the major findings of the current study is; force of mortality is higher for female in earlier age compare to male (Table 4). The crossover of force of mortality occurs approximately at age 15 for, and it remained almost equal at the adolescent age for both sexes. After the age 20 and onwards, the force of mortality is lower in case of female compare to male.

Fig.1: Estimated force of mortality for males and females (Matlab HDSS-2010).



Discussion

The explanation of observed age patterns of mortality with mathematical models is one of the oldest and most important topics in demography. Infant and child mortality are decreasing in Bangladesh in a notable trend, however, future increases in life expectancy will therefore required additional reductions in adult mortality. This study examines the age pattern of adult mortality and discusses the pattern of force of mortality from a demographic surveillance area.

Data of Matlab HDSS is used in this study to estimate the force of mortality for both sexes. It is seen that number of survivors (l_x) for male and female fits a 3rd degree polynomial well, that is a cubic polynomial model. The obtained results showed that though the age-specific death rates follow almost U-shaped pattern but the force of mortality is different than that. Decreasing trend in observed up to adolescent age, almost constant pattern is observed up to age 35 and increasing pattern is observed after age 40, which has a rapid increase after age 55 year and so on. Life expectancy increases for Matlab for the last few decades, but still the early mortality is available, which is reflected in pattern of force of mortality. Female force of mortality is lower than male from adolescent period, which is another sign of improvement of public health sector of Matlab. We hope these latest findings on mortality studies, would encourage the government and non-government organizations (NGOs) and planners to plan to strengthen the socio-economic development and health care program.

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